

## ULTRA- AND MICROFILTRATION IN A CELL WITHOUT MIXING

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UDC 541.18.045

*The process of ultrafiltration in a cell without mixing with allowance for the diffusion coefficient as a function of the concentration is described.*

When medical and microbiological investigations are performed the need to concentrate high-molecular-weight substances from a small volume of the solution arises. In particular, the method of membrane filters is widely used in counting viable microorganisms contained in water. Based on the size of the microorganism we can always select a membrane that retains it and larger microorganisms on its surface completely. In the process of concentration, a gel-like layer of high-molecular-weight substances forms in the membrane region, i.e., a dynamic membrane forms. Here, it is undesirable to interfere with the process of gelation by different physical actions and by mixing the water in the cell since the additional mechanical actions can impair the viability of the microorganisms, thus decreasing their actual number. As a result the process has a pronounced nonstationary character, and the description of special features of mass transfer in time and the estimation of the duration of the process itself for a certain amount of the solution for prescribed properties of the membrane are of practical interest. A qualitatively similar situation can occur in a flow membrane channel with a shaped surface if, for some reason, a stagnation zone forms in a well or a recess.

Results of theoretical investigation of similar problems are presented in a number of works [1, 2]. However, in [1] the solution of the problem is obtained based on the method of integral relations, whose accuracy depends strongly on the choice of the shape of the parameterized profile of the sought function, while in [2] the boundary condition for the growth of the gel layer is written incorrectly. Furthermore, in the general case, the diffusion coefficient depends on the concentration [3-5].

The production-type cell that contains the unmixed solution has the shape of a truncated cone of height  $H$  with apex angle in the cone's axial cross section  $\alpha$ . In this case, the height of the solution in the process of ultrafiltration as a function of time will obey the relation

$$\frac{h(\tau)}{H} = \begin{cases} 1 + H \tan \alpha + \frac{H^2}{3} \tan^2 \alpha - \frac{V\tau}{H}, & \tau \leq \tau_1, \\ 1 + H \tan \alpha + \frac{H^2}{3} \tan^2 \alpha - \frac{V\tau_1}{H} - \int_{\tau_1}^{\tau} \frac{V\delta}{H} d\tau, & \tau \geq \tau_1. \end{cases} \quad (1)$$

Here the first equation holds before the process of gelation begins, while the second equation holds when a gel layer forms and grows on the membrane surface. The time  $\tau_1$  is determined from the condition of attaining the gelation concentration at the membrane surface, i.e.,

$$\Theta = \Theta_g \quad (\tau = \tau_1). \quad (2)$$

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Academic Scientific Complex "A. V. Luikov Institute of Heat and Mass Transfer of the National Academy of Sciences of Belarus," Minsk. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 71, No. 5, pp. 771-774, September-October, 1998. Original article submitted June 12, 1997.

Let us allow for the diffusion coefficient as a function of the concentration of the dissolved substance and assume that the membrane has ideal selectivity, i.e., retains completely the molecules of the dissolved substance. Then the process will be described by a diffusion-convection equation that, in dimensionless form, will be represented as

$$\frac{\partial \Theta}{\partial \tau} - V_{\delta} \frac{\partial \Theta}{\partial \eta} = \frac{\partial}{\partial \eta} \frac{1}{\text{Pe}_0} \left[ \frac{1}{D(\Theta)} \frac{\partial \Theta}{\partial \eta} \right], \quad (3)$$

$$\text{Pe}_D = D_D / V_0 r = \text{Pe}_0 D(\Theta), \quad D_D = D_0 D(\Theta).$$

We assume a hyperbolic dependence of the diffusion coefficient on the concentration:

$$D(\Theta) = \frac{1}{1 + aC_0\Theta}, \quad (4)$$

where  $a$  can be both positive [3] and negative [4] and has dimensions that are inverse to the dimensions of the concentration in the initial solution.

Before the beginning of gelation on the membrane we should assume  $V_{\delta} = 1$  in Eq. (3) and solve it under the boundary conditions

$$\Theta + \frac{1}{\text{Pe}_0 D(\Theta)} \frac{\partial \Theta}{\partial \eta} = 0, \quad (\eta = 0), \quad \Theta(0, \eta) = 1, \quad \Theta(\tau, \Delta) = 1. \quad (5)$$

Once the gel layer forms on the membrane the boundary conditions are transformed to the form

$$\Theta_g \frac{\partial \delta}{\partial \tau} = \Theta_g V_{\delta} + \frac{1}{\text{Pe}_0 D(\Theta_g)} \frac{\partial \Theta}{\partial \eta} \Big|_{\eta = \delta}, \quad \Theta(\tau, \delta) = \Theta_g, \quad (6)$$

$$\Theta(\tau, \Delta) = 1, \quad V_{\delta} = \frac{1}{1 + k\delta}, \quad \delta(\tau_1) = 0.$$

To solve the systems of equations (3), (5) and (3), (6), let us use the methodology presented in [6]. Then, for the time of the beginning of gelation  $\tau_1$ , we obtain the relation

$$\tau_1 = \frac{1}{aC_0 \text{Pe}_0} \left[ \ln \Theta_g - (1 + aC_0) \ln \frac{(1 + aC_0) \Theta_g}{1 + aC_0 \Theta_g} \right] \quad (a \neq 0) \quad (7)$$

and for the specific capacity of the membrane  $V_{\delta}$  in gel-layer formation and growth the formula

$$V_{\delta} = \left[ \frac{1 + 2(\tau/\tau_1 - 1)}{1 + (2\Theta_g - 1)/k\tau_1} \right]^{1/2} \quad (\tau \geq \tau_1). \quad (8)$$

In the denominator of Eq. (8), the term in the parentheses characterizes the resistance of the gel layer to the transmembrane flux  $V$ . Omitting unimportant details of calculating the integral in Eq. (1), we find

$$\frac{h}{H} = 1 + H \tan \alpha + \frac{H^2}{3} \tan^2 \alpha - \frac{2\Theta_g - 1}{k} \left( \frac{1}{V_{\delta}} - 1 \right) - \frac{\tau_1}{V_{\delta}}. \quad (9)$$

Formula (9) holds when the thickness of the solution  $h$  is not smaller than the total thickness of the gel and the diffusion layer:

$$h/H \geq \Delta = \frac{1}{k} \left( \frac{1}{V_{\delta}} - 1 \right) + \frac{1}{\text{Pe}_0 V_{\delta}} \ln \frac{(1 + aC_0) \Theta_g}{1 + aC_0 \Theta_g}. \quad (10)$$

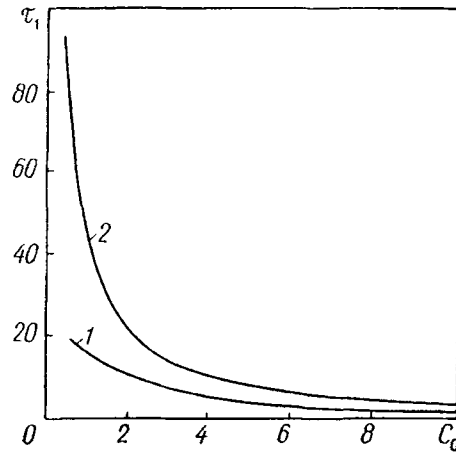


Fig. 1. Time of gelation in emptying a cone-shaped funnel vs. the initial concentration ( $D_0 = 7.1 \cdot 10^{-11} \text{ m}^2/\text{sec}$ ,  $a = 0.12$ ): 1)  $\tau_1$  by (7), 2)  $\tau_1$  by (14).

The time  $\tau_2$  at which there will be just the gel and solution of diffusion-layer thickness in the production-type unmixed cell is determined from Eq. (8) if for  $V_\delta$  we use the formula

$$V_\delta = \left( \frac{2\Theta_g}{k} + \frac{1}{aC_0 Pe_0} \ln \frac{1 + aC_0 \Theta_g}{1 + aC_0} \right) / \left( 1 + H \tan \alpha + \frac{H^2}{3} \tan^2 \alpha - \frac{2\Theta_g}{k} \right), \quad (11)$$

which follows from relations (7), (9), and (10). Although the concentration gradient of a high-molecular-weight substance inside the diffusion layer can be very significant, the layer itself is very thin and its effect can be disregarded, i.e., the solution of the diffusion problem can be extended to include the gel layer. In this case, in the time  $\tau = \tau_3$  the solution is completely filtered, after which a gel layer of thickness

$$\delta = \frac{h}{H} = \frac{(V/V_{\delta_3} - 1)}{k} \quad (12)$$

remains in the cell. To estimate the emptying time for the cell  $\tau_3$ , we can proceed from formula (8) with a known value of  $V_{\delta_3}$ . The unknown value of  $V_{\delta_3}$  is easy to determine from relations (9) and (12):

$$V_{\delta_3} = \left( \frac{2\Theta_g}{k} + \tau_1 \right) / \left( 1 + H \tan \alpha + \frac{H^2}{3} \tan^2 \alpha - \frac{2\Theta_g}{k} \right). \quad (13)$$

Let us determine the effect of the diffusion coefficient as a function of the concentration on the process of ultrafiltration in the cell. We use experimental data on a solution of bovine serum albumin [4] that, to a satisfactory degree of accuracy, can be approximated by the hyperbolic dependence

$$D_D = 7.1 \cdot 10^{-11} / (1 + 0.12C_0\Theta).$$

The need to allow for  $D$  as a function of the concentration will be shown by the example of calculating the time of the beginning of gelation. If the diffusion coefficient is independent of the concentration, then

$$\tau_1 = \frac{1}{Pe_0} (\Theta_g - \ln \Theta_g - 1). \quad (14)$$

Figure 1 gives  $\tau_1$  as a function of the initial concentration for a gelation concentration of 50 g/mole. Curve 1 is calculated by (7). If we assume  $D = \text{const}$  and, from experimental data, determine the constant as the integral mean (see (14)),  $\tau_1$  as a function of the initial concentration will be represented by curve 2. Their comparison shows that in describing ultra- and microfiltration processes we should allow for the diffusion coefficient as a

function of the concentration. Thus, the above formulas enable us to calculate the rate of transmembrane transfer  $V_\delta$  as a function of time and to determine the duration of the process of solution filtration.

The work was carried out with support from the Fund for Fundamental Research of the Republic of Belarus (project T94-020 of January 27, 1995).

## NOTATION

$x$  and  $y$ , coordinate axes, the origin of the coordinate system coincides with the center of the lower orifice of the cone-shaped funnel, the  $y$  axis coincides with the cone height;  $R$ , radius of the funnel's lower orifice, m;  $\eta = y/R$ , dimensionless coordinate;  $V_0$ , rate of transmembrane transfer before the gel layer forms, m/sec ( $V_0 = f(\Delta P, \mu, \xi_0)$ , where  $\mu$  is the dynamic viscosity of the solution,  $\Delta P$  is the pressure difference, and  $\xi_0$  is the coefficient of resistance of the membrane);  $C_1$ , concentration of the dissolved substance in the solution to be separated, g/mole;  $D_0$ , diffusion coefficient of the initial solution, m<sup>2</sup>/sec;  $\tau_1 = t_1 V_0/R$ , dimensionless time of the beginning of gelation;  $h = \hat{h}/R$ , dimensionless height of the solution in the funnel;  $H = \hat{H}/R$ , dimensionless height of the funnel;  $C_0$ , percentage of the dissolved substance in the initial solution;  $\Theta = C/C_1$ , dimensionless concentration;  $\Theta_g$ , dimensionless concentration of gelation;  $Pe_D = V_0 R/D_D = \hat{V}_0 R/D_0 D(\Theta)$ , diffusion Peclet number;  $\Delta$ , dimensionless total thickness of the gel and the diffusion layer;  $V_\delta = \hat{V}_\delta/V_0$ , dimensionless rate of transmembrane transfer under conditions of gel-layer buildup.

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